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Gravitational Phase Transition of Heavy Neutrino Matter

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Abstract

We study the phase transition of a system of self-gravitating neutrinos in the presence of a large radiation density background in the framework of the Thomas-Fermi model. We show that, by cooling a non-degenerate gas of massive neutrinos below some critical temperature, a condensed phase emerges, consisting of quasi-degenerate supermassive neutrino stars. These compact dark objects could play an important role in structure formation in this universe, as they might in fact provide the seeds for galactic nuclei and quasi-stellar objects.

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A gas of massive fermions which interact only gravitationally has interesting thermal properties which may have important consequences for the early universe. The canonical and grand canonical ensembles for such a system have been shown to have a nontrivial thermodynamical limit [1, 2]. Under certain conditions these systems will undergo a phase transition that is accompanied by gravitational collapse [3]. This phase transition occurs uniquely for the attractive gravitational interaction of neutral fermions. There is indeed no such phase transition in the case of charged fermions [4]. Of course, gravitational condensation will also take place if the fermions have an additional weak interaction, as neutrinos, neutralinos, axions and other weakly interacting massive particles generally do. Moreover, it is important to note that the phase transition will occur, irrespective of the magnitude of the initial density fluctuations in the fermion gas, and, as we shall shortly demonstrate, irrespective of the amount of background radiation. To be specific, we henceforth assume that this neutral fermion is the heaviest neutrino ν_τ, although this is not essential for most of the subsequent discussion.

The ground state of a gravitationally condensed neutrino cloud, with mass below the Chandrasekhar limit, is a cold neutrino star [5, 6, 7, 8], in which the degeneracy pressure balances the gravitational attraction of the neutrinos. Degenerate stars of neutrinos in the mass range between $m_\nu = 10$ and 25 keV are particularly interesting [8], as they could explain, without resorting to the black hole hypothesis, at least some of the features that are observed around supermassive compact dark objects, which are reported to exist at the centers of a number of galaxies [9, 10, 11, 12, 13, 14] including our own [15, 16] and quasi-stellar objects (QSO) [17, 18, 19, 20]. Indeed, there is little difference between a supermassive black hole and a neutrino star of the same mass near the Chandrasekhar limit, a few Schwarzschild radii away from the object.

The existence of a quasi-stable neutrino in this mass range is neither ruled out by particle and nuclear physics experiments nor by direct astrophysical observations [8]. In the early universe, however, it would lead to an early neutrino matter dominated phase some time after nucleosynthesis and prior to recombination. In such a universe, the microwave background temperature would be reached much too early to accommodate the oldest stars in globular clusters, cosmochronology and the Hubble expansion age, if the Standard Model of Cosmology is correct. However, the early universe might have evolved quite differently in the presence of such a heavy neutrino. In particular, it is conceivable that primordial neutrino stars have been formed in local condensation processes during a gravitational phase transition that must have occurred some time between nucleosynthesis and recombination. Aside from reheating the gaseous phase of heavy neutrinos, the latent heat produced by the condensed phase might have contributed partly to reheating the radiation as well. Moreover, the bulk part of the heavy neutrinos (and antineutrinos) will have annihilated efficiently into light neutrinos via the Z^0 in the interior of these supermassive neutrino stars [5, 6, 7, 8]. Since both these processes will increase the age of the universe, or the time when the universe reaches today's microwave background temperature [8], it does not seem excluded that a quasi-stable massive neutrino in the mass range between 10 and 25 keV is compatible with the cosmological observations [5, 8].

The purpose of this paper is to study the formation of such a neutrino star during a gravitational phase transition in an expanding universe at the time when the energy densities of neutrino matter and radiation are of comparable magnitude.

We assume here that the equilibrium distribution of the ν_τ gas is spherically symmetric and the energy density ρ_γ of the radiation background homogeneous. At this stage ρ_γ consists of photons and the two remaining relativistic neutrino species ν_μ and ν_e, is given by

$$\rho_\gamma = \frac{a}{2} g_2 T_\gamma^4, \tag{1}$$

with

$$g_N = 2 + \frac{7}{4}\left(\frac{4}{11}\right)^{4/3} N. \tag{2}$$

The gravitational potential $V(r)$ satisfies the Poisson equation

$$\Delta V = 4\pi G m_\nu (m_\nu n_\nu + \rho_\gamma), \tag{3}$$

where the number density of τ neutrinos (including antineutrinos) of mass m_ν can be expressed in terms of the Fermi-Dirac distribution at a finite temperature T as

$$n_\nu(r) \ = \ \frac{g_\nu}{4\pi^2} (2 m_\nu T)^{3/2} I_{\frac{1}{2}} \left(\frac{\mu - V(r)}{T}\right), \tag{4}$$

with

$$I_n(\eta) \ = \ \int_0^\infty \frac{\xi^n d\xi}{1 + e^{\xi - \eta}}. \tag{5}$$

g_ν denotes the combined spin degeneracy factors of neutrinos and antineutrinos (i.e. g_ν is 2 or 4 for Majorana or Dirac neutrinos respectively), and μ is the chemical potential. It is convenient to introduce the normalized reduced potential

$$v = \frac{r}{m_\nu G M_\odot}(\mu - V), \tag{6}$$

$M_\odot$ being the solar mass, and dimensionless variable $x = r/R_0$ with the scale factor

$$R_0 = \left(\frac{3\pi}{4\sqrt{2} m_\nu^4 g_\nu G^{3/2} M_\odot^{1/2}}\right)^{2/3} = 2.1377 \ \text{lyr} \left(\frac{17.2 \ \text{keV}}{m_\nu}\right)^{8/3} g_\nu^{-2/3}. \tag{7}$$

Eq.(3) then takes the simple form

$$\frac{1}{x}\frac{d^2 v}{dx^2} = -\frac{3}{2}\beta^{-3/2} I_{\frac{1}{2}}\left(\beta\frac{v}{x}\right) - 4\pi \frac{R_0^3 \rho_\gamma}{M_\odot}, \tag{8}$$

where we have introduced the normalized inverse temperature defined as

$$\beta = T_0/T; \qquad T_0 = m_\nu G M_\odot / R_0. \tag{9}$$

At zero temperature we recover from eq.(8) the well-known Lané-Emden differential equation [6,8]

$$\frac{d^2v}{dx^2} = -\frac{v^{3/2}}{\sqrt{x}}. \tag{10}$$

The solution of the differential equation (8) requires boundary conditions. We assume here that the neutrino gas is enclosed in a spherical cavity of radius R corresponding to $x_1 = R/R_0$. We further require the total neutrino mass to be M_ν, and the total radiation mass within the cavity to be M_γ, and we allow for the possibility of a pointlike mass M_C at the origin, which could be e.g. a compact seed of other exotic matter. $v(x)$ and $v(x)$ is then related to its derivative at $x = x_1$ by

$$v'(x_1) = \frac{1}{x_1}\left(v(x_1) - \frac{M_C + M_\gamma + M_\nu}{M_\odot}\right), \tag{11}$$

which in turn is related to the chemical potential by $\mu = T_0 v'(x_1)$. $v(x)$ at $x = 0$ is related to the point mass at the origin by $M_C/M_\odot = v(0)$.

Similar to the case of the Lané-Emden equation, it is easy to show that eq.(8) has a scaling property: if $v(x)$ is a solution of eq.(8) at a temperature T and a cavity radius R, then $\tilde{v}(x) = A^3 v(Ax)$ with $(A > 0)$ is also a solution at the temperatures $\tilde{T} = A^4 T$, $\tilde{T}_\gamma = A^4 T_\gamma$ and the cavity radius $\tilde{R} = R/A$.

It is important to note that only those solutions that minimize the free energy are physical. The free energy functional is defined as [2]

$$\begin{aligned} F[n] &= \mu[n]N_\nu - W[n] \\ &- Tg_\nu \int \frac{d^3r d^3p}{(2\pi)^3} \ln(1 + \exp\left(-\frac{p^2}{2m_\nu T} - \frac{V[n]}{T} + \frac{\mu[n]}{T}\right)), \end{aligned} \tag{12}$$

where

$$V[n] = -Gm_\nu \int d^3r' \frac{m_\nu n(r') + \rho_\gamma}{|\mathbf{r} - \mathbf{r}'|}, \tag{13}$$

and

$$W[n] = -\frac{1}{2}Gm_\nu^2 \int d^3r d^3r' \frac{n(r)n(r')}{|\mathbf{r} - \mathbf{r}'|}. \tag{14}$$

The chemical potential in eq. (12) varies with density so that the number of neutrinos $N_\nu = M_\nu/m_\nu$ is kept fixed.

All the relevant thermodynamical quantities such as number density, pressure, free energy, energy and entropy can be expressed in terms of v/x

$$n_\nu(x) = \frac{M_\odot}{m_\nu R_0^3}\frac{3}{8\pi}\beta^{-3/2} I_{\frac{1}{2}}\left(\beta\frac{v}{x}\right), \tag{15}$$

$$P_\nu(x) = \frac{M_\odot T_0}{m_\nu R_0^3}\frac{3}{8\pi}\beta^{-5/2} I_{\frac{3}{2}}\left(\beta\frac{v}{x}\right) = \frac{2}{3}\varepsilon_{\text{kin}}(x), \tag{16}$$

$$
\begin{aligned}
F &= \frac{1}{2}\mu(N_\nu + N_\gamma) + \frac{3}{5}M_\gamma^2\frac{1}{R} \\
&+ \frac{1}{2}T_0R_0^3\int d^3x(n_\nu - n_\gamma)\frac{v(x)-v(0)}{x} - R_0^3\int d^3xP_\nu(x),
\end{aligned} \tag{17}
$$

$$
\begin{aligned}
E &= \frac{1}{2}\mu(N_\nu + N_\gamma) + \frac{3}{5}M_\gamma^2\frac{1}{R} \\
&- \frac{1}{2}T_0R_0^3\int d^3x\left[(n_\nu + n_\gamma)\frac{v(x)}{x} + (n_\nu - n_\gamma)\frac{v(0)}{x}\right] + R_0^3\int d^3x\varepsilon_{\mathrm{kin}}(x),
\end{aligned} \tag{18}
$$

$$
S = \frac{1}{T}(E - F). \tag{19}
$$

In eqs. (17) and (18) we have introduced the effective radiation number $N_\gamma = M_\gamma/m_\nu$, and the effective radiation number density $n_\gamma = \rho_\gamma/m_\nu$.

We now turn to the numerical study of a system of self-gravitating massive neutrinos with arbitrarily chosen total mass $M = 10M_\odot$ varying the cavity radius R and the radiation mass M_γ. Owing to the scaling properties, the system may be rescaled to any physically interesting mass. For definiteness, the ν_τ mass is chosen as $m_\nu = 17.2$ keV which is about the central value of the mass region between 10 and 25 keV [8] that is interesting for our scenario.

In fig. 1 we present our results for a gas of neutrinos in a cavity of radius $R = 100R_0$, and with no background radiation i. e. with $M_\gamma = 0$. We find the three distinct solutions in the temperature interval $T = (0.049 - 0.311)T_0$ of which only two are physical, namely those for which the free-energy assumes a minimum. The density distributions, corresponding to such two solutions are shown in the first plot in fig. 1. We refer to the solution that exists above the mentioned interval at arbitrary high temperature as “gas”, while the solution which exists at low temperatures and eventually becomes a degenerate Fermi gas at $T = 0$, we refer to as “condensate”.

In fig. 1 we also plot various extensive thermodynamical quantities (per neutrino) as functions of neutrino temperature. The phase transition takes place at the point T_t where the free energy of the gas and condensate become equal. The transition temperature $T_t = 0.19442T_0$ is indicated by the dotted line in the free energy plot. The top dashed curve in the same plot corresponds to the unphysical solution. At $T = T_t$ the energy and the entropy have a discontinuity.

Next we study a system of massive neutrino gas in a radiation background. In the course of universe expansion, the heavy neutrino becomes nonrelativistic at a time t_{NR} corresponding to the temperature $T_{NR} = m_\nu$. At that time the radiation dominates the matter by about a factor of 15. A neutrino cloud with the total mass e.g. $10^9M_\odot$, which is by about factor of 10 below the Chandrasekhar limit for $m_\nu = 17.2$ keV and $g_\nu = 4$, would have a radius

$$
R_{NR} = 2[G(M_\nu + M_\gamma)t_{NR}^2]^{1/3}, \tag{20}
$$

yielding $R_{NR} = 0.265$ light days. As we shall shortly see this is way below the critical value and therefore there exists only one solution for any fixed temperature. As the

universe expands and cools down the relative amount of radiation mass decreases as

$$M_\gamma = 15 M_\nu \frac{R_{NR}}{R}. \qquad (21)$$

Concerning the R dependence of the temperature we assume that during the radiation domination the temperature T of the neutrino gas is fixed by the radiation heat bath so that it continues to decrease as $1/R$ until the neutrino matter starts to dominate. At that point the system being in gaseous phase will have the entropy per particle $S/N_\nu = 7.60$. From then on, the T will not be coupled to the radiation heat bath anymore and will decrease with R according to the adiabatic expansion of the neutrino matter itself.

Shortly after the matter starts to dominate, we reach the region of instability and the first order phase transition takes place. Fig 2. shows how the entropy behaves around the critical region. We find as we decrease the radius that the first order phase transition becomes weaker and eventually disappears at $R = R_c = 5.34$ light days corresponding to the critical temperature $T_c = 0.0043 m_\nu$ and the radiation mass relative to the neutrino mass $M_\gamma = 0.67 M_\nu$. This is the critical point of a second order phase transition. This behavior is typical for a mean-field type of models [21].

The initial entropy of 7.6 per neutrino drops to 0.75 so that the latent heat per particle yields $\Delta E = T\Delta S = 0.016 m_\nu$. Thus, the condensate formation is accompanied by a release of considerable amount of energy which will reheat the radiation environment.

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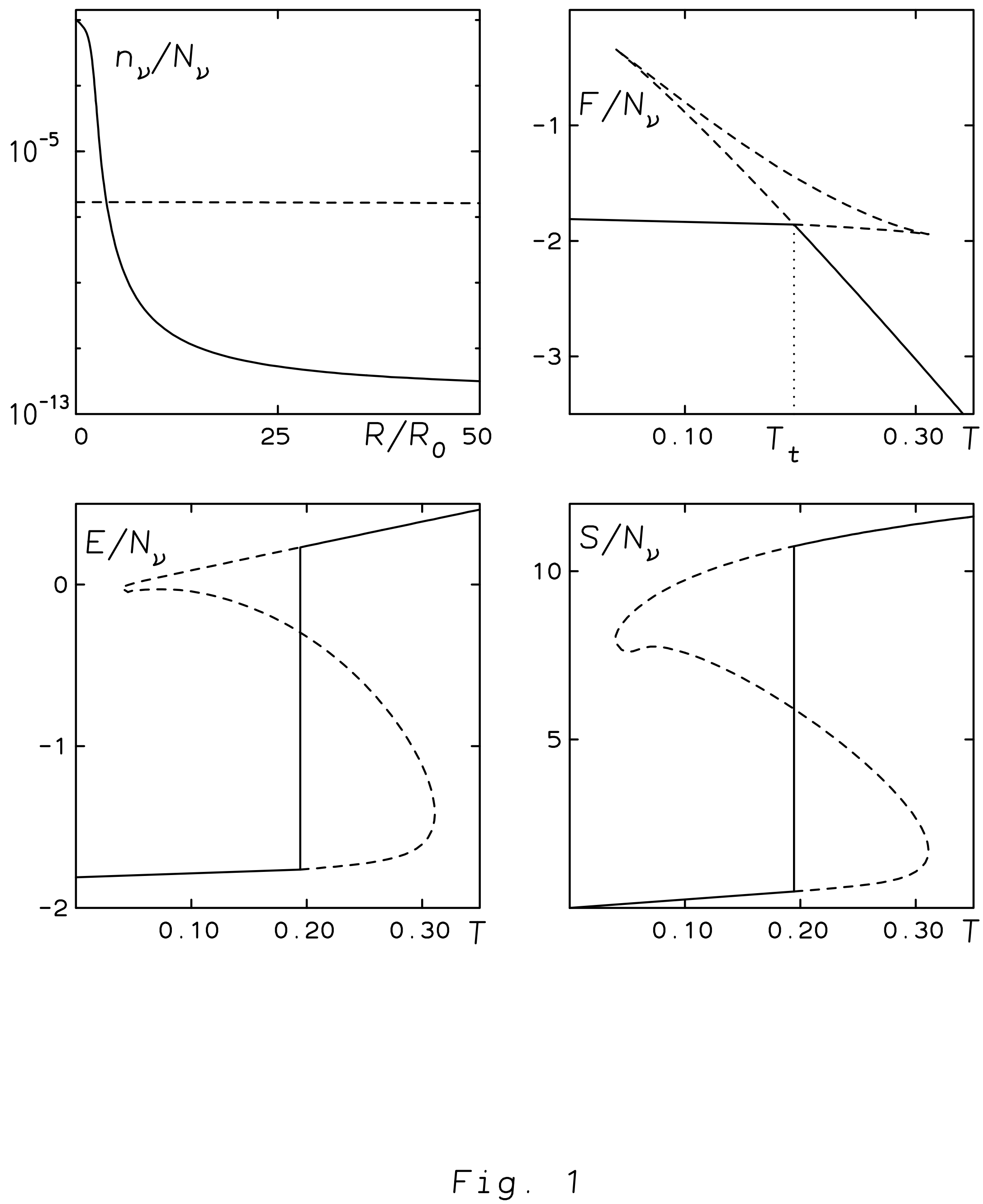
Figure Captions:

1: Density distribution normalized to unity for condensate-like (solid line) and gas-like (dashed line) solutions at $T = T_t$. Free energy, energy and entropy per particle as a function of temperature. Temperature, energy and free energy, are in units of T_0.

2: Entropy per particle as a function of temperature for various radii (in light days) of the neutrino cloud with $M_\nu = 10^9 M_\odot$.



n_ν/N_ν
10^-5
10^-13
0
25
R/R_0
50
F/N_ν
-1
-2
-3
0.10
T_t
0.30
T
E/N_ν
0
-1
-2
0.10
0.20
0.30
T
S/N_ν
10
5
0.10
0.20
0.30
T

Fig. 1

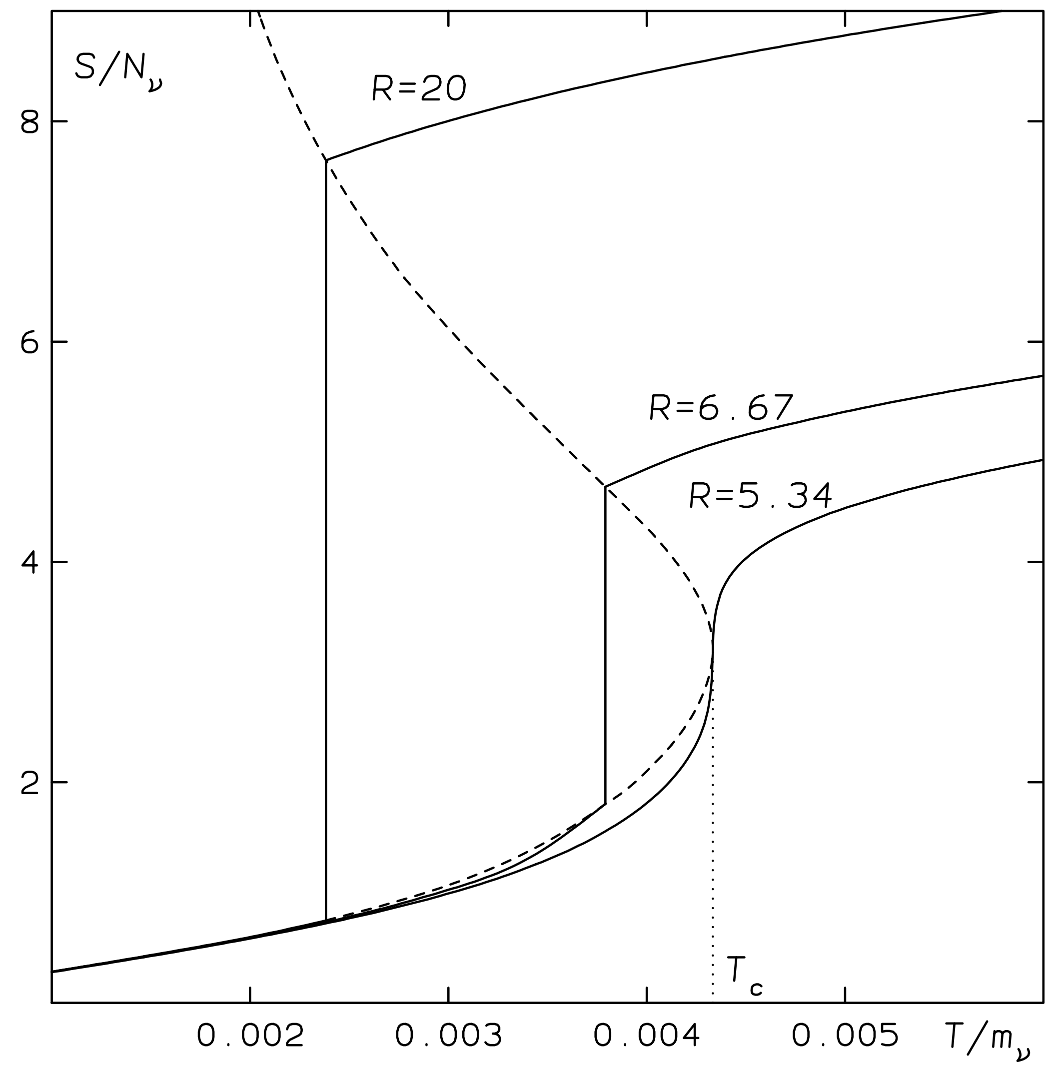


Fig. 2